

000000 000000 TTTTTTTTTT SSSSSSSS PPPPPPPP
000000 000000 TTTTTTTTTT SSSSSSSS PPPPPPPP
00 00 TT SS PP PP 00 00 WW WW DDDDDDDD
00 00 TT SS PP PP 00 00 WW WW DDDDDDDD
00 00 TT SS PP PP 00 00 WW WW DDDDDDDD
00 00 TT SS PP PP 00 00 WW WW DDDDDDDD
00 00 TT SS PPPPPP 00 00 WW WW DDDDDDDD
00 00 TT SS PPPPPP 00 00 WW WW DDDDDDDD
00 00 TT SS PP 00 00 WW WW DDDDDDDD
00 00 TT SS PP 00 00 WW WW DDDDDDDD
00 00 TT SS PP 00 00 WW WW DDDDDDDD
00 00 TT SS PP 00 00 WW WW DDDDDDDD
000000 000000 TT SSSSSSSS PP 000000 WW WW DDDDDDDD
000000 000000 TT SSSSSSSS PP 000000 WW WW DDDDDDDD
000000 000000 TT SSSSSSSS PP 000000 WW WW DDDDDDDD

LL IIIII SSSSSSSS
LL IIIII SSSSSSSS
LL IIIII SS SS
LL IIIII SS SS
LL IIIII SSSSSS SSSSSS
LL IIIII SS SS
LL IIIII SS SS
LLLLLLLLLL IIIII SSSSSSSS
LLLLLLLLLL IIIII SSSSSSSS

(2)	58	HISTORY	: Detailed current edit history
(2)	88	DECLARATIONS	
(4)	220	OTSSPOWDD - DOUBLE to DOUBLE giving DOUBLE result	

```
0000 1 .TITLE OTSSPOWDD - DOUBLE PRECISION ** DOUBLE PRECISION power routine
0000 2 .IDENT /2-007/ ; File: OTSPOWDD.MAR Edit: JCW2007
0000 3
0000 4
0000 5 ****
0000 6 *
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0000 24 *
0000 25 *
0000 26 ****
0000 27
0000 28
0000 29 : FACILITY: Language support library - user callable
0000 30 ++
0000 31 : ABSTRACT:
0000 32
0000 33     1) Double base to double power.
0000 34     2) Floating base to double power.
0000 35     3) Double base to floating power.
0000 36
0000 37     Floating overflow can occur
0000 38     Undefined exponentiation can occur if:
0000 39         1) Negative base
0000 40         2) 0 base and power is 0 or negative.
0000 41
0000 42
0000 43
0000 44 --
0000 45
0000 46 : VERSION: 2
0000 47
0000 48 : HISTORY:
0000 49
0000 50 : AUTHOR:
0000 51     Bob Hanek, 3-Mar-83: Version 2
0000 52
0000 53 : MODIFIED BY:
0000 54
0000 55
0000 56 :
```

0000 58 .SBTTL HISTORY ; Detailed current edit history
0000 59
0000 60
0000 61 : Edit history for Version 2 of OTSSPOWDD
0000 62
0000 63 : 2-001 Implemented new algorithm. RNH 3-Mar-83
0000 64 : 2-002 The code would overflow when underflow was the proper result. This
0000 65 : error arose from the ADDF3 #^X4DC0, R2, R6 code, which rounded the
0000 66 : result so that when SUBF #^X4DC0, R6 was performed the value in
0000 67 : R6 was greater than R2, causing R0 to be positive when it should have
0000 68 : been negative. The bugfix was to change the test for overflow from
0000 69 : R0 to R6 (both of these should be of the same sign). JCW 13-MAY-83.
0000 70 : 2-003 Change INDEX table to be local instead of global. Also change
0000 71 : the instruction format from INDEX(Rx) to INDEX[Rx] to avoid linker
0000 72 : errors. LEB 25-May-1983
0000 73 : 2-004 Change remaining table references to use [Rx] instead of (Rx).
0000 74 : Make A1_TABLE and A2_TABLE local symbols instead of globals.
0000 75 : LEB 26-May-1983
0000 76 : 2-005 Change leftover table reference to use [Rx] instead of (Rx).
0000 77 : LEB 29-May-1983
0000 78 : 2-006 Added two ROTL #-3,Rx,Rx instructions to scale the value of Rx back
0000 79 : from 'index*2^3' to 'index' before A_TABLE[Rx] is referenced. The
0000 80 : INDEX was not scaled back to yield values of 'index' instead of
0000 81 : 'index*2^3' because the mathematics of the code used does need the
0000 82 : value of index*2^3 in several computations. JCW 7-Jun-1983
0000 83 : 2-007 Corrected a bug involving a SYS F FLTOVF F error during a MULD R6, R2.
0000 84 : Code was added to see if a MTH overflow message or a zero should be
0000 85 : returned. JCW 19-Jan-1984
0000 86 :
0000 87 :

```
0000 88 .SBTTL DECLARATIONS
0000 89
0000 90 ; INCLUDE FILES:
0000 91
0000 92
0000 93
0000 94
0000 95
0000 96 ; EXTERNAL SYMBOLS:
0000 97
0000 98 .DSABL GBL
0000 99 .EXTRN MTH$$SIGNAL ; Math error routine
0000 100 .EXTRN MTHSK_FLOOVEMAT ; Floating overflow code
0000 101 .EXTRN MTHSK_FLOUNDMAT ; Floating underflow code
0000 102 .EXTRN MTHSK_UNDEXP ; Undefined exponentiation code
0000 103
0000 104 ; MACROS:
0000 105
0000 106 ; MACROS:
0000 107 $SFDEF ; Define stack frame symbols
0000 108
0000 109 ; EQUATED SYMBOLS:
0000 110
00000004 0000 111 base = 4 ; base input formal - by-value
0000000C 0000 112 exp = 12 ; exponent input formal - by-value
00000008 0000 113 fexp = 8 ; exponent when base is floating
000007FC 0000 114 ACMASK = ^M< R2, R3, R4, R5, R6, R7, R8, R9, R10>
0000 115 ; register saving mask
```

```

0000 117 : PSECT DECLARATIONS:
0000 118 : PSECT _OTSSCODE      PIC,SHR,QUAD,EXE,NOWRT
0000 120 : .PSECT _OTSSCODE      PIC,SHR,QUAD,EXE,NOWRT
0000 121 : : program section for OTSS code
0000 122 : CONSTANTS:
0000 123 : 
0000 124 : 
0000 125 : 
0000 126 : 
0000 127 : The INDEX table gives the appropriate byte offset into the A1 and A2 Tables.
0000 128 : 
0000 129 : 
08 08 08 08 08 08 00 00 00 00 0000 130 INDEX: .BYTE ^X00, ^X00, ^X00, ^X08, ^X08, ^X08, ^X08, ^X08
18 10 10 10 10 10 10 08 0008 131 .BYTE ^X08, ^X10, ^X10, ^X10, ^X10, ^X10, ^X10, ^X18
20 20 20 18 18 18 18 18 0010 132 .BYTE ^X18, ^X18, ^X18, ^X18, ^X18, ^X18, ^X20, ^X20
28 28 28 28 20 20 20 20 0018 133 .BYTE ^X20, ^X20, ^X20, ^X20, ^X28, ^X28, ^X28, ^X28
30 30 30 30 30 30 28 28 0020 134 .BYTE ^X28, ^X28, ^X30, ^X30, ^X30, ^X30, ^X30, ^X30
38 38 38 38 38 38 30 30 0028 135 .BYTE ^X30, ^X30, ^X38, ^X38, ^X38, ^X38, ^X38, ^X38
40 40 40 40 40 40 40 38 0030 136 .BYTE ^X38, ^X40, ^X40, ^X40, ^X40, ^X40, ^X40, ^X40
48 48 48 48 48 48 48 40 0038 137 .BYTE ^X40, ^X48, ^X48, ^X48, ^X48, ^X48, ^X48, ^X48
50 50 50 50 50 50 50 48 0040 138 .BYTE ^X48, ^X50, ^X50, ^X50, ^X50, ^X50, ^X50, ^X50
58 58 58 58 58 58 50 50 0048 139 .BYTE ^X50, ^X50, ^X58, ^X58, ^X58, ^X58, ^X58, ^X58
60 60 60 60 60 58 58 58 0050 140 .BYTE ^X58, ^X58, ^X58, ^X60, ^X60, ^X60, ^X60, ^X60
68 68 68 68 60 60 60 60 0058 141 .BYTE ^X60, ^X60, ^X60, ^X60, ^X68, ^X68, ^X68, ^X68
70 70 68 68 68 68 68 68 0060 142 .BYTE ^X68, ^X68, ^X68, ^X68, ^X68, ^X68, ^X70, ^X70
70 70 70 70 70 70 70 70 0068 143 .BYTE ^X70, ^X70, ^X70, ^X70, ^X70, ^X70, ^X70, ^X70
78 78 78 78 78 78 78 78 0070 144 .BYTE ^X78, ^X78, ^X78, ^X78, ^X78, ^X78, ^X78, ^X78
80 80 80 80 80 78 78 78 0078 145 .BYTE ^X78, ^X78, ^X78, ^X80, ^X80, ^X80, ^X80, ^X80
0080 146 
0080 147 
0080 148 .ALIGN QUAD
0080 149 
0080 150 : 
0080 151 : For k = 0, 1, ..., 16, the k-th entry of the A1 table is 2^(k/16) rounded
0080 152 : to 56 bits and the k-th entry of the A2 table 2^(k/16) - A1_TABLE(k)
0080 153 : rounded to 56 bits.
0080 154 : 
0080 155 : 
00000000 00004080 0080 156 A1_TABLE:
487B67CC AAC34085 0088 157 .QUAD ^X0000000000004080
8BD7E3EA 95C1408B 0090 158 .QUAD ^X487B67CCAAAC34085
11C373AB C3D34091 0098 159 .QUAD ^X8BD7E3EA95C1408B
B8A9518D 37F04098 00A0 160 .QUAD ^X11C373ABC3D34091
A1126091 F532409E 00A8 161 .QUAD ^XB8A9518D37F04098
5139A9B1 FED640A5 00B0 162 .QUAD ^XA1126091F532409E
A14BEA42 583E40AD 00B8 163 .QUAD ^X5139A9B1FED640A5
DE6533F9 04F340B5 00C0 164 .QUAD ^XA14BEA42583E40AD
OC379F58 08A340BD 00C8 165 .QUAD ^XDE6533F904F340B5
06DB1155 672A40C5 00D0 166 .QUAD ^XOC379F5808A340BD
8481151F 248C40CE 00D8 167 .QUAD ^X06DB1155672A40C5
9D6BCAD6 44FC40D7 00E0 168 .QUAD ^X8481151F248C40CE
94E1EC2A CCDE40E0 00E8 169 .QUAD ^X9D6BCAD644FC40D7
2439E7DD C0C640EA 00F0 170 .QUAD ^X94E1EC2ACCDE40E0
86CC1524 257D40F5 00F8 171 .QUAD ^X2439E7DDC0C640EA
00000000 00004100 0100 172 .QUAD ^X86CC1524257D40F5
00000000 00004100 0100 173 .QUAD ^X0000000000004100

```

0108	174				
00000000 00000000	0108	175 A2_TABLE:			
0AA6DC61 2E4A2326	0110	176 .QUAD	^X0000000000000000		
3AD6EBC5 20DCA348	0118	177 .QUAD	^X0AA6DC612E4A2326		
B96B6382 3F5B23D8	0120	178 .QUAD	^X3AD6EBC520DCA348		
5C864630 8D5A245E	0128	179 .QUAD	^XB96B63823F5B23D8		
288CC1EC BEDDA424	0130	180 .QUAD	^X5C8646308D5A245E		
477C04EF 1A14A32F	0138	181 .QUAD	^X288CC1ECBEDDA424		
29975CDC D9FDA3E6	0140	182 .QUAD	^X477C04EF1A14A32F		
156BEC99 4D04A477	0148	183 .QUAD	^X29975CDCD9FDA3E6		
25E1C093 AEFDA402	0150	184 .QUAD	^X156BEC994D04A477		
6C8ABCDA 0754A38B	0158	185 .QUAD	^X25E1C093AEFDA402		
506AE383 EE53A360	0160	186 .QUAD	^X6C8ABCDA0754A38B		
FD2C4466 6597A2BC	0168	187 .QUAD	^X506AE383EE53A360		
6E9AA824 32C42308	0170	188 .QUAD	^XF2C44666597A2BC		
C1B9D6CD 40B523BB	0178	189 .QUAD	^X6E9AA82432C42308		
B66031EB EE7423B1	0180	190 .QUAD	^XC1B9D6CD40B523BB		
00000000 00000000	0188	191 .QUAD	^XB66031EBEE7423B1		
	0190	192 .QUAD	^X0000000000000000		
	0190	193 .QUAD	^X0000000000000000		
00000000 00002500	0190	194 TWO_M55:.QUAD	^X0000000000002500		
	0198	195			
17F1295C AA3B44B8	0198	196 C: .QUAD	^X17F1295CAA3B44B8		
00002000 AA3B44B8	01A0	197 C1: .QUAD	^X00002000AA3B44B8		
E8800BBB C17F3695	01A8	198 C2: .QUAD	^XE8800BBC17F3695		
	01B0	199			
973BB7EA 01121CC2	01B0	200 LOGTAB: .QUAD	^X973BB7EA01121CC2		
2742C3D0 CA802581	01B8	201 .QUAD	^X2742C3DOCA802581		
F145FD62 19A02E3D	01C0	202 .QUAD	^XF145FD6219A02E3D		
1DA4FD64 FE9F3723	01C8	203 .QUAD	^X1DA4FD64FE9F3723		
00000000 00000000	01D0	204 .QUAD	^X0000000000000000		
	01D8	205 LOGLEN = <.-LOGTAB>/8			
	01D8	206			
	01D8	207			
	01D8	208 EXPTAB:			
0D2F11A3 E6F41FFF	01D8	209 .QUAD	^X0D2F11A3E6F41FFF	:	0.27094695278739704E-19
2D1FB089 85242521	01E0	210 .QUAD	^X2D1FB08985242521	:	0.35024086731698588E-16
6DC43BFA C3FF2A2E	01E8	211 .QUAD	^X6DC43BFAC3FF2A2E	:	0.3805761563274748E-13
D9857D84 955B2F1D	01F0	212 .QUAD	^XD9857D84955B2F1D	:	0.35830323049894278E-10
0627B825 584633E3	01F8	213 .QUAD	^X0627B825584633E3	:	0.26466421444330986E-07
2C9CFC16 FDEF3875	0200	214 .QUAD	^X2C9CFC16FDEF3875	:	0.14662262387640433E-04
CF7AF7D1 72173CB1	0208	215 .QUAD	^XCF7AF7D172173CB1	:	0.54152123481245728E-02
00000000 00000000	0210	216 .QUAD	^X0000000000000000	:	0.0
	0218	217 EXPLEN = <.-EXPTAB>/8			
	0218	218			

0218 220 .SBTTL OTSSPOWDD - DOUBLE to DOUBLE giving DOUBLE result
0218 221
0218 222 :++
0218 223 : FUNCTIONAL DESCRIPTION:
0218 224
0218 225 OTSSPOWDR - DOUBLE result = DOUBLE base ** FLOATING exponent
0218 226 OTSSPOWRD - DOUBLE result = FLOATING base ** DOUBLE exponent
0218 227 OTSSPOWDD - DOUBLE result = DOUBLE base ** DOUBLE exponent
0218 228
0218 229 The DOUBLE result is given by:
0218 230
0218 231 base exponent result
0218 232 -----
0218 233
0218 234 = 0 > 0 0.0
0218 235 = 0 = 0 Undefined Exponentiation
0218 236 = 0 < 0 Undefined Exponentiation
0218 237
0218 238 < 0 any Undefined Exponentiation
0218 239
0218 240 > 0 > 0 $2^{(\exp * \text{LOG2}(base))}$
0218 241 > 0 = 0 1.0
0218 242 > 0 < 0 $2^{(\exp * \text{LOG2}(base))}$
0218 243
0218 244
0218 245 Floating Overflow and Underflow can occur.
0218 246 Undefined Exponentiation can occur if:
0218 247 1) base is 0 and exponent is 0 or negative
0218 248 2) base is negative
0218 249
0218 250 The basic approach to computing $x^{**}y$ as $2^{[y*\text{log2}(x)]}$ is the following:
0218 251
0218 252 Step 1: Compute $\text{log2}(x)$ to sufficient precision to guarantee an
0218 253 accurate final result (see below.)
0218 254 Step 2: Compute $y*\text{log2}(x)$ to at least the accuracy that $\text{log2}(x)$
0218 255 was computed.
0218 256 Step 3: Evaluate $2^{[y*\text{log2}(x)]}$ accurate to the precision of the
0218 257 datatype in question.
0218 258
0218 259 To determine the accuracy to which $\text{log2}(x)$ must be computed to, write
0218 260 $y*\text{log2}(x)$ as $I + h$, where I is the integer closest to $y*\text{log2}(x)$, and
0218 261 $h = y*\text{log2}(x) - I$ (Note that $|h| < 1/2$.) Then
0218 262
0218 263 $2^{[y*\text{log2}(x)]} = 2^I * (2^h)$.
0218 264
0218 265 Since the factor 2^I can be incorporated into the final result by an integer
0218 266 addition to the exponent field, we can assume that the multiplication by
0218 267 2^I incurs no error. Thus the total error in the final result is determined
0218 268 by how accurately 2^h can be computed. If the final result has p fraction
0218 269 bits, we would like h to have at least p good bits. In fact it would be
0218 270 nice if h had a few extra guard bits, say 4. Consequently, we would like
0218 271 h to be accurate to $p + 4$ bits.
0218 272
0218 273 Let e be the number of bits allocated to the exponent field of the data type
0218 274 in question. If I requires more than e bits to represent, then overflow or
0218 275 underflow will occur. Therefore if the product $y*\text{log2}(x)$ has $e + p + 4$ good
0218 276 bits, the final result will be accurate. This requires that $\text{log2}(x)$ be

0218 277 : computed to at least p + e + 4 bits.
 0218 278 :
 0218 279 : Since log2(x) must be computed to more bits of precision than is available
 0218 280 : in the base data type, either the next level of precision or multi-precision
 0218 281 : arithmetic must be used. We begin by writing
 0218 282 :
 0218 283 :
 0218 284 :
 0218 285 : $\log_2(x) = \log_2(b) + \sum_{n=0}^{\infty} c(2n+1) * z^n$,
 0218 286 :
 0218 287 :
 0218 288 : Where $c(1) = 1$, and $z' = (2/\ln 2)[(z-b)/(z+b)]$. Hence
 0218 289 :
 0218 290 :
 0218 291 :
 0218 292 :
 0218 293 :
 0218 294 :
 0218 295 :
 0218 296 :
 0218 297 :
 0218 298 :
 0218 299 : Note that if $p(z')$ is computed to p bits, and $\log_2(b) + z'$ is computed
 0218 300 : to $p+e+4$ bits and overhangs $p(z')$ by $e+4$ bits, the required accuracy will
 0218 301 : be achieved. Consequently, the essential tricks, are to pick b such that
 0218 302 : the overhang can be achieved and to compute $\log_2(b) + z'$ to $p + e + 4$ bits.
 0218 303 :
 0218 304 :
 0218 305 : CALLING SEQUENCE:
 0218 306 :
 0218 307 : power.wd.v = OTSSPOWDR (base.rd.v, exponent.rf.v)
 0218 308 : power.wd.v = OTSSPOWRD (base.rf.v, exponent.rd.v)
 0218 309 : power.wd.v = OTSSPOWDD (base.rd.v, exponent.rd.v)

INPUT PARAMETERS:

Base and exponent parameters are call by value

IMPLICIT INPUTS:

none

OUTPUT PARAMETERS:

none

IMPLICIT OUTPUTS:

none

FUNCTIONAL VALUE:

OTSSPOWDR - DOUBLE base ** FLOATING power
 OTSSPOWRD - FLOATING base ** DOUBLE power
 OTSSPOWDD - DOUBLE base ** DOUBLE power

SIDE EFFECTS:

SIGNALS MTHSK_FLOOVEMAT if floating overflow.
 SIGNALS MTHSK_FLOUNDMAT if floating underflow.
 SIGNALS MTHS_UNDEXP (82 = 'UNDEFINED EXPONENTIATION') if
 1) Base is 0 and exponent is 0 or negative

OTSSPOWDD
2-007

- DOUBLE PRECISION ** DOUBLE PRECISION ^{N 12} p 16-SEP-1984 01:57:20 VAX/VMS Macro V04-00
OTSSPOWDD - DOUBLE to DOUBLE giving DOUB 6-SEP-1984 11:28:03 [MTHRTL.SRC]OTSSPOWDD.MAR;1 Page 8
(4)

0218 334 :
0218 335 :
0218 336 :
0218 337 :--

2) base is negative

OT
1-

	07FC	0218	339	.ENTRY OTSSPOWDR, ACMASK	;
		021A	340	SUBL #20, SP	; Allocate 5 longwords on the stack
04 AE	5E 14	C2 02	021A 341	MOVL #2, (SP)	; Set number of arguments equal to 2
0C AE	6E 02	D0 70	021D 342	MOVD base(AP), base(SP)	; Move base to stack
0000024C'EF	04 AC	OC AC	0220 343	CVTFD exp(AP), exp(SP)	; convert FLOATING exponent to DOUBLE
		56 FA	0225 344	CALLG (SP), OTSSPOWDD	; Call OTSSPOWDD
		04 02	022A 345	RET	
			0231 346		
			0232 347		
			0232 348		
	07FC	0232	349	.ENTRY OTSSPOWRD, ACMASK	;
		0234	350	SUBL #20, SP	; Allocate 5 longwords on the stack
04 AE	5E 14	C2 02	0234 351	MOVL #2, (SP)	; Set number of arguments equal to 2
0C AE	6E 02	D0 56	0237 352	CVTFD base(AP), base(SP)	; convert FLOATING base to DOUBLE
0000024C'EF	04 AC	08 AC	023A 353	MOVD fexp(AP), exp(SP)	; Move exponent to stack
			023F 354	CALLG (SP), OTSSPOWDD	; Call OTSSPOWDD
			0244 355	RET	
			024B 356		
			024C 357		

```

024C 359
07FC 024C 360 .ENTRY OTSSPOWDD, ACMASK ;
024E 361 :
024E 362 : Move x to R0/R1. If x < 0, or x = 0 and y <= 0, return "UNDEFINED"
024E 363 : EXPONENTIATION" error condition, otherwise attempt to compute x**y
024E 364 :
024E 365 :
024E 366 GET_BASE:
50 04 AC 70 024E 367 MOVD base(AP), R0 ; R0/R1 <-- x
0252 368 COMMON: 369 BGTR DEFINED ; If x > 0 attempt to compute x**y
0252 370 BLSS UNDEFINED ; Branch to error code for x < 0
06 19 0254 371 TSTD exp(AP) ; Check sign of y (Note that x = 0)
0256 372 BLEQ UNDEFINED ; Branch to error condition if y <= 0
01 15 0259 025B 373
025B 374 :
025B 375 : If processing continues here, this implies that x = 0 and y > 0. Return
025B 376 : x**y = 0
025B 377 :
025B 378 04 025B 379 RET ; Return
025C 380
025C 381 :
025C 382 : If processing continues here, this implies that an undefined exponentiation
025C 383 : was attempted. Signal error and return
025C 384 :
025C 385 025C 386 UNDEFINED:
50 8000 8F 3C 025C 387 MOVZWL #^X8000, R0 ; R0/R1 <-- Reserved operand
51 D4 0261 388 CLRL R1
00000000'GF 01 FB 0263 389 MOVZBL #MTHSK UNDEXP, -(SP) ; Put error code on stack
0267 390 CALLS #1, G^MTH$$$SIGNAL ; Convert error number to 32 bit
026E 391 condition code and signal error.
026E 392 NOTE: Second argument is not re-
026E 393 quired since there is no JSB entry.
026E 394 04 026E 395 RET ; Return
026F 396 :
026F 397 : If processing continues here will attempt to compute x**y as 2^[y*log2(x)].
026F 398 : We begin by determining k and f such that x = 2^k*f, where 1 <= f < 2.
026F 399 :
026F 400 026F 401 DEFINED:
54 50 FFFF807F 8F CB 026F 402 BICL3 #^xFFFF807F, R0, R4 ; R4 <-- 2^7*(biased exponent of x)
54 00004080 8F C2 0277 403 SUBL #^X4080, R4 ; R4 <-- 2^7*k = 2^7*(exponent of x - 1)
50 54 C2 027E 404 SUBL R4, R0 ; R0 <-- f = 2*(fraction field of x)
0281 405
0281 406 :
0281 407 : We are now ready to compute log2(x). This computation is based on the
0281 408 : following identity:
0281 409 :
0281 410 :
0281 411 : log2(2^k*f) = k + log2(f) +  $\frac{2}{\ln(2)} \frac{1}{2j+1} z^{(2j+1)}$ , where z =  $\frac{f-a}{f+a}$ .
0281 412 :
0281 413 :
0281 414 : We begin by determining a as b^i, where b = 2^(1/16) and i is
0281 415 : between 0 and 16 inclusive. Specifically i is chosen by table look-up

```

0281 416 ; in such a fashion as to minimize the magnitude of z. Since log2(a) = i/16
 0281 417 ; we may write
 0281 418 ;
 0281 419 ; $\log_2(x) = k + i/16 + z*p(z*z)$.
 0281 420 ;
 0281 421 ;
 0281 422 EVAL_LOG2:
 SA 50 FFFFFF80 8F CB 0281 423 BICL3 #^FFFFFF80, R0, R10 ; R10 <- index to INDEX table
 5A FD72 CF4A 90 0289 424 MOVB INDEX[R10], R10 ; R10 <- $i \cdot 2^3$
 7E 54 5A C1 028F 425 ADDL3 R10, R4, -(SP) ; SP --> $2^7 * (k + i/16)$
 5A 5A FD 8F 9C 0293 426 ROTL #3, R10, R10 ; R10 <- i
 0298 427 ; R10 will be multiplied by 2^3 by
 0298 428 ; table references like the line below.
 0298 429 ; The linker will cause an error if
 0298 430 ; () are used instead of [] for these
 0298 431 ; table references.
 0298 432 ;
 0298 433 ;
 0298 434 ; We proceed by computing $z = (f-a)/(f+a)$. In order to insure the accuracy of
 0298 435 ; the final result, it is necessary to compute z to at least 68 bits. Since no
 0298 436 ; back up data type is available, we must compute z in two parts: $z = z_1 + z_2$,
 0298 437 ; where z_1 is the high 24 bits of z and z_2 is the low 56 bits of z. Further,
 0298 438 ; to obtain the desired accuracy it is necessary to work with $a = a_1 + a_2$,
 0298 439 ; where a_1 and a_2 are the high and low 56 bits respectively of 'a'. We begin
 0298 440 ; computing (in single precision)
 0298 441 ;
 0298 442 ; $z_1 = (f - a_1)/(f + a_1)$
 0298 443 ;
 0298 444 ; Note that $f - a_1$ can be computed exactly in 56 bits, but $f + a_1$ may require 57
 0298 445 ; bits. The 57 bit can be determined by the exclusive or of the low bits of
 0298 446 ; f and a_1 .
 0298 447 ;
 0298 448 ;
 58 FDE3 CF4A 7D 0298 449 MOVQ A1_TABLE[R10], R8 ; R8/R9 <- a_1
 7E 51 59 CD 029E 450 XORL3 R9, R1, -(SP) ; SP --> XOR of low bits of a_1 and x
 02A2 451 ; (This will be used to determine the
 02A2 452 ; 57 bit of $f + a_1$.)
 52 50 58 61 02A2 453 ADDD3 R8, R0, R2 ; R2/R3 <- $f + a_1$ (rounded)
 50 58 62 02A6 454 SUBD R8, R0 ; R0/R1 <- $f - a_1$ (exact)
 58 50 52 47 02A9 455 DIVF3 R2, R0, R8 ; R8 <- z_1 (single)
 59 00 D0 02AD 456 MOVL #0, R9 ; R8/R9 <- z_1 (double)
 02B0 457 ;
 02B0 458 ; To compute z_2 we note
 02B0 459 ;
 02B0 460 ; $z = z_1 + z_2 = (f - a_1 - a_2)/(f + a_1 + a_2)$
 02B0 461 ;
 02B0 462 ; $\Rightarrow z_2 = (f - a_1 - a_2)/(f + a_1 + a_2) - z_1$
 02B0 463 ;
 02B0 464 ; Now let $v = f + a_1 + a_2 = v_1 + v_2$, where v_1 and v_2 are the high 24 and low
 02B0 465 ; 56 bits of v respectively. Then
 02B0 466 ;
 02B0 467 ; $z_2 = [(f - a_1 - z_1 \cdot v_1) - (a_2 + z_1 \cdot v_2)]/v$
 02B0 468 ;
 02B0 469 ; We begin by computing v_1 and $f - a_1 - z_1 \cdot v_1$
 02B0 470 ;
 02B0 471 ;
 54 52 D0 02B0 472 MOVL R2, R4 ; R4 <- high longword of $f + a_1$

56 55 00 D0 02B3 473 MOVL #0, R5 ; R4/R5 <- v1
 52 54 63 02B6 474 SUBD3 R4, R2, R6 ; R6/R7 <- f + a1 - v1 (exact)
 54 58 64 02BA 475 MULD R8, R4 ; R4/R5 <- z1*v1 (exact)
 50 54 62 02BD 476 SUBD R4, R0 ; R0/R1 <- f - a1 - z1*v1 (exact)
 02C0 477 :
 02C0 478 : Compute v2 and a2 + a1*v2
 02C0 479 :
 54 FE43 CF4A 7D 02C0 480 MOVQ A2_TABLE[R10], R4 ; R4/R5 <- a2
 FFFFEEEE 8F CA 02C6 481 BICL #^XFFFFFFFFFF, (SP) ; Check if w was rounded
 05 13 02CD 482 BEQL 1S ; Branch if not rounded
 56 FEBD CF 62 02CF 483 1S: SUBD TWO_M55, R6 ; Correct for rounding error (exact)
 56 54 60 02D4 484 ADDD R4, R6 ; R6/R7 <- v2
 56 58 64 02D7 485 MULD R8, R6 ; R6/R7 <- z1*v2
 56 54 60 02DA 486 ADDD R4, R6 ; R6/R7 <- a2 + z1*v2
 02DD 487 :
 02DD 488 : Compute z2
 02DD 489 :
 50 56 62 02DD 490 SUBD R6, R0 ; R0/R1 <- (f-a1-z1*v1)-(a2-z1*v2)
 50 52 66 02E0 491 DIVD R2, R0 ; R0/R1 <- z2
 02E3 492 :
 02E3 493 : The next step is to compute log2(x) accurate to at least 68 bits. This is
 02E3 494 : accomplished as follows, let
 02E3 495 :
 02E3 496 : $w = 2^{^7} \log_2(x)$
 02E3 497 : = $(2^{^7})[k + i/16 + z*p(z*z)]$
 02E3 498 : = $2^{^7}*(k + i/16) + (2^{^7})*z*(c_0 + c_2*z^2 + \dots + c_{10}*z^{10})$
 02E3 499 : = $[2^{^7}*(k + i/16) + z'] + z'(d_2*z'^2 + \dots + d_{10}*z'^{10})$
 02E3 500 : = $[2^{^7}*(k + i/16) + z'] + z'*q(z'*z')$
 02E3 501 : = w1 + w2
 02E3 502 :
 02E3 503 : where $z' = (2^{^7}*c_0)*z$ and w1 and w2 are the high 24 and low 56 bits of w
 02E3 504 : respectively. Note that the choice of 'a' used in computing z, guarantees
 02E3 505 : that z' overhangs $z'*q(z'*z')$ by at least 13 bits. Hence, if w is computed
 02E3 506 : as w1 + w2, the necessary 68 bits of accuracy can be obtained. $c_0 = 2^{^6}/\ln(2)$.
 02E3 507 :
 02E3 508 : We begin by defining
 02E3 509 :
 02E3 510 : c = high 56 bits of $(2^{^7}*c_0)$
 02E3 511 : c1 = high 28 bits of $(2^{^7}*c_0)$
 02E3 512 : c2 = low 56 bits of $(2^{^7}*c_0)$
 02E3 513 : then
 02E3 514 : $z' = (z_1 + z_2)*(c_1 + c_2)$
 02E3 515 : = $z_1*c_1 + z_1*c_2 + z_2*c$.
 02E3 516 :
 02E3 517 :
 54 58 FEB9 CF 65 02E3 518 MULD3 C1, R8, R4 ; R4/R5 <- c1*z1
 58 FEBB CF 64 02E9 519 MULD C2, R8 ; R8/R9 <- c2*z1
 50 FEA6 CF 64 02EE 520 MULD C, R0 ; R0/R1 <- c*z2
 50 58 60 02F3 521 ADDD R8, R0 ; R0/R1 <- c*z2 + c2*z1
 58 50 54 61 02F6 522 ADDD3 R4, R0, R8 ; R8/R9 <- z'
 02FA 523 :
 02FA 524 : We proceed by letting
 02FA 525 :
 02FA 526 : and
 02FA 527 : w1 = high 24 bits of $2^{^7}*(k + i/16) + z1*c1$
 02FA 528 :
 02FA 529 : w2' = $\{[2^{^7}*(k + i/16) + z1*c1 - w1] + z1*c2\} + z2*c$.

02FA 530 :
 02FA 531 :
 02FA 532 :
 02FA 533 :
 02FA 534 :
 02FA 535 :
 02FA 536 :
 02FA 537 :
 02FA 538 :
 02FA 539 :
 5A 04 AE 4E 02FA 540 CVTLF 4(SP), R10 ; R10 <- $2^7(k + i/16)$
 56 54 5A 41 02FE 541 ADDF3 R10, R4, R6 ; R6 <- $2^7(k+i/16) + z1*c1$ in single
 57 00 00 00 0302 542 MOVL #0, R7 ; R6/R7 <- w1
 52 56 5A 43 0305 543 SUBF3 R10, R6, R2 ; R2 <- bits of $z1*c1$ included in w1
 0309 544
 53 00 00 00 0309 545 MOVL #0, R3 ; = $-[2(k+i/16)+z1*c1 - w1]$
 54 52 62 030C 546 SUBD R2, R4 ; Convert R2 to double
 6E 50 54 61 030F 547 ADDD3 R4, R0, (SP) ; $[2^7(k+i/16)+z1*c1 - w1] + c1*z1$
 0313 548
 0313 549 :
 0313 550 : Compute w2
 0313 551 :
 0313 552 :
 FE93 50 58 58 65 0313 553 MULD3 R8, R8, R0 ; R0/R1 <- $z'*z'$
 CF 04 50 75 0317 554 POLYD R0, #LOGLEN-1, LOGTAB ; R0/R1 <- $q(z'*z')$
 50 58 64 031D 555 MULD R8, R0 ; R0/R1 <- $z'*q(z'*z')$
 50 6E 60 0320 556 ADDD (SP), R0 ; R0/R1 <- w2
 0323 557
 0323 558 :
 0323 559 : We now calculate $y*\log2(x) = (y1+y2)*(w1+w2) = y1*w1 + y2*w1 + y*w2$, where
 0323 560 : y1 and y2 are the high and low 28 bits of y respectively.
 0323 561 :
 54 0C AC 7D 0323 562 MOVQ exp(AP), R4 ; R4/R5 <- y
 0327 563
 0327 564
 0327 565 :
 0327 566 : Test for the possibility of overflow in the computation of y*w1.
 0327 567 : This will occur if the exponent of y plus the exponent of w1 is greater
 0327 568 : than 127.
 0327 569 :
 0327 570 :
 52 54 08 07 EF 0327 571 EXTZV #7, #8, R4, R2 ; biased exp of y
 52 0080 8F A2 032C 572 SUBW2 #^X80, R2 ; unbiased exp of y
 53 56 08 07 EF 0331 573 EXTZV #7, #8, R6, R3 ; biased exp of w1
 53 0080 8F A2 0336 574 SUBW2 #^X80, R3 ; unbiased exp of w1
 53 52 A0 033B 575 ADDW2 R2, R3 ; unbiased exp of w1*y
 53 007F 8F B1 033E 576 CMPW #^X7F, R3 ; largest unbiased exp possible is 127
 76 19 0343 577 BLSS Y_TIMES_W1_OVER
 50 54 64 0345 578 MULD R4, R0 ; R0/R1 <- y*w2
 52 54 D0 0348 579 MOVL R4, R2 ; R2 <- high longword of y1
 53 55 FFFF0FFF 8F CB 034B 580 BICL3 #^XFFFF0FFF, R5, R3 ; R2/R3 <- y1
 54 52 62 0353 581 SUBD R2, R4 ; R4/R5 <- y2
 52 56 64 0356 582 MULD R6, R2 ; R2/R3 <- y1*w1
 54 56 64 0359 583 MULD R6, R4 ; R4/R5 <- y2*w1
 035C 584
 035C 585 :
 035C 586 : The next step in computing $2^{[y*\log2(x)]}$ is to write $y*\log2(x)$ as

035C 587 :
 035C 588 :
 035C 589 :
 035C 590 : where I is an integer, j is an integer between 0 and 15 inclusive, and
 035C 591 : g is a fraction in the interval [-1/2, 1/2)
 035C 592 :
 035C 593 :
 56 52 00004DC0 8F 41 035C 594 ADDF3 #^X4DC0, R2, R6 ; 3*2^5 is used in this truncation process
 0364 595 : to avoid a possible normalization
 0364 596 : that could occur if the number is neg
 56 00004DC0 8F 42 0364 597 SUBF #^X4DC0, R6
 57 00 D0 036B 598 MOVL #0, R7
 52 56 62 036E 599 SUBD R6, R2
 54 52 60 0371 600 ADDD R2, R4
 50 54 60 0374 602 ADDD R4, R0
 57 56 49 0377 603 CVTFW R6, R7
 35 1D 037A 604 BVS EXCEPTION_1 ; R0/R1 <- g
 037C 605 : R7 <- 2^7*(I + j/16) in integer
 037C 606 : Branch if |I| is too large
 037C 607 : We can now compute
 037C 608 :
 037C 609 : $x^{**}y = 2^{[y * \log_2(x)]} = 2^{[I + j/16 + g/16]}$
 037C 610 :
 037C 611 : $= (2^I) * [A * (B+1)] = 2^I * [A + A * B]$, where
 037C 612 :
 037C 613 : A = $2^{(j/16)}$ is obtained by table look-up and B = $2^{(g/16)} - 1$ is obtained
 037C 614 : by a Min/Max approximation.
 037C 615 :
 037C 616 :
 FE56 CF 07 50 75 037C 617 POLYD R0, #EXPLEN-1, EXPTAB ; R0 <- B = $2^{(g/16)} - 1$. (Chebyshev
 52 57 FFFFFFF80 8F CB 0382 618 polynomial approx. of degree 5)
 52 52 FD 8F 9C 0382 619 BICL3 #^XFFFFFF80, R7, R2 ; R2 <- $2^3 * \text{index into A1_TABLE}$
 50 FCEC CF42 64 038A 620 ROTL #3, R2, R2
 50 FD6E CF42 60 0395 621 MULD A1_TABLE[R2], R0
 50 FCEO CF42 60 039B 622 ADDD A2_TABLE[R2], R0
 57 007F 8F AA 03A1 623 ADDD A1_TABLE[R2], R0
 50 57 A0 03A6 624 BICW #^X7F, R7
 007F 8F 50 B1 03A9 625 ADDW R7, R0
 07 15 03AE 626 CMPW R0, #^X7F
 04 03B0 627 BLEQ EXCEPTION_2 ; test for over/underflow
 03B1 628 RETURN: RET ; see what exception is if neg or = 0
 03B1 629 : otherwise return result in R0
 03B1 630 :
 03B1 631 : Handlers for software detected over/underflow conditions follow
 03B1 632 :
 03B1 633 :
 03B1 634 EXCEPTION 1:
 56 53 03B1 635 TSTF R6 ; if big ARG > 0 goto overflow
 1D 18 03B3 636 BGEQ OVER ; handler, otherwise go to
 08 11 03B5 637 BRB UNDER ; underflow handler
 03B7 638 :
 57 B5 03B7 639 EXCEPTION 2:
 17 18 03B9 640 TSTW R7 ; test sign of I; if I < 0
 03BB 641 BGEQ OVER ; go to overflow handler
 03BB 642 :
 03BB 643 :

 OTS Sym
 BAS
 CHF
 CHF
 CHF
 CHF
 CON
 DO
 DO
 DO
 EXC
 EXP
 EXP
 MTH
 MTH
 MTH
 MTH
 OTS
 PAR
 POW
 PSL
 SET
 SF\$
 SF\$
 SQU
 SQU
 SSS
 SSS
 SSS
 SSS
 UND
 PSE ---

 \$AB _OT
 Pha ---
 Ini
 Com
 Pas
 Sym
 Pas
 Sym
 Pse
 Cro

```

      03BB 644 ; y*w1 would have caused a hardware system floating overflow error. If y<0,
      03BB 645 ; then we should return a result of 0 since result = 2^(y*(w1+w2)). Note,
      03BB 646 ; y can not be zero.
      03BB 647 ;
      03BB 648
      03BB 649 Y_TIMES_W1_OVER:
      54 73 03BB 650 TSTD R4 ; if y < 0 no overflow is needed
      13 14 03BD 651 BGTR OVER ; overflow
      03BF 652
      03BF 653 :
      03BF 654 : Underflow; if user has FU set, signal error. Always return 0.0
      03BF 655 :
      03BF 656
      03BF 657 UNDER:
      OB 04 AD 50 7C 03BF 658 CLRQ R0 ; R0/R1 <-- 0
      06 E1 03C1 659 BBC #6, SF$W_SAVE_PSW(FP), 2$ ; has user enabled floating underflow?
      7E 00'8F 01 9A 03C6 660
      00000000'GF 01 FB 03CA 661 MOVZBL #MTH$K_FLOUNDMAT, -(SP) ; Put underflow code on stack
      03D1 662 CALLS #1, G^MTH$$SIGNAL ; Convert to MTH$_FLOUNDMAT (32-bit
      03D1 663 ; VAX-11 exception code) and
      03D1 664 ; signal condition
      04 03D1 665 2$: RET ; return
      03D2 666
      03D2 667 :
      03D2 668 : Signal floating overflow, return reserved operand, -0.0
      03D2 669 :
      03D2 670
      03D2 671 OVER:
      50 7E 00'8F 9A 03D2 672 MOVZBL #MTH$K_FLOOVEMAT, -(SP) ; else process for overflow
      01 0F 79 03D6 673 ASHQ #15, #T, R0 ; Put overflow code on stack
      03DA 674 ; R0 = result = reserved operand
      03DA 675 ; -0.0. R0 will be copied to
      03DA 676 ; signal mechanism vector (CHF$L_MCH_R0/R1)
      03DA 677 ; so can be fixed up by any error
      03DA 678 ; handler
      00000000'GF 01 FB 03DA 679 CALLS #1, G^MTH$$SIGNAL ; signal condition
      04 03E1 679 RET ; return - R0 restored from CHF$L_MCH_R0/R1
      03E2 680
      03E2 681 .END
  
```

★ ★ F

A1_TABLE	000000080	R	02
A2_TABLE	00000108	R	02
ACMASK	= 000007FC		
BASE	= 00000004		
C	00000198	R	02
C1	000001A0	R	02
C2	000001A8	R	02
COMMON	00000252	RR	02
DEFINED	0000026F	RR	02
EVAL_LOG2	00000281	R	02
EXCEPTION_1	000003B1	R	02
EXCEPTION_2	000003B7	R	02
EXP	= 0000000C		
EXPLEN	= 00000008		
EXPTAB	000001D8	R	02
FEXP	= 00000008		
GET_BASE	0000024E	R	02
INDEX	00000000	R	02
LOGLEN	= 00000005		
LOGTAB	000001B0	R	02
MTH\$SIGNAL	*****		X 00
MTH\$K_FLOOVEMAT	*****		X 00
MTH\$K_FLOUNDMAT	*****		X 00
MTH\$K_UNDEXP	*****		X 00
OTSSPOWDD	0000024C	RG	02
OTSSPOWDR	00000218	RG	02
OTSSPOWRD	00000232	RG	02
OVER	000003D2	R	02
RETURN	000003B0	R	02
SFSW_SAVE_PSW	= 00000004		
TWO_A55	00000190	R	02
UNDEFINED	0000025C	R	02
UNDER	000003BF	R	02
Y_TIMES_W1_OVER	000003BB	R	02

+-----+
! Psect synopsis !
+-----+

PSECT name

Allocation	PSECT No.	Attributes												
00000000	(0.)	00	(0.)	NOPI	USR	CON	ABS	LCL	NOSHR	NOEXE	NORD	NOWRT	NOVEC	BYTE
00000000	(0.)	01	(1.)	NOPI	USR	CON	ABS	LCL	NOSHR	EXE	RD	WRT	NOVEC	BYTE
000003E?	(994.)	02	(2.)	PIC	USR	CON	REL	LCL	SHR	EXE	RD	NOWRT	NOVEC	QUAD

! Performance indicators !

Phase

Page faults	CPU Time	Elapsed Time
32	00:00:00.10	00:00:00.63
125	00:00:00.75	00:00:06.82
134	00:00:02.35	00:00:06.76
0	00:00:00.05	00:00:00.10
130	00:00:01.51	00:00:04.55
4	00:00:00.06	00:00:00.54

Psect synopsis output 1 00:00:00.02 00:00:00.02
Cross-reference output 0 00:00:00.00 00:00:00.00
Assembler run totals 428 00:00:04.85 00:00:19.43

The working set limit was 1200 pages.

12217 bytes (24 pages) of virtual memory were used to buffer the intermediate code.
There were 10 pages of symbol table space allocated to hold 62 non-local and 2 local symbols.
741 source lines were read in Pass 1, producing 21 object records in Pass 2.
9 pages of virtual memory were used to define 8 macros.

+-----+
! Macro library statistics !
+-----+

Macro library name -----

_S255\$DUA28:[SYSLIB]STARLET.MLB;2

Macros defined -----

4

88 GETS were required to define 4 macros.

There were no errors, warnings or information messages.

MACRO/ENABLE=SUPPRESSION/DISABLE=(GLOBAL,TRACEBACK)/LIS=LISS:OTSPOWDD/OBJ=OBJ\$:OTSPOWDD MSRC\$:MTHJACKET/UPDATE=(ENH\$:MTHJACKET)+MSRC

0264 AH-BT13A-SE
VAX/VMS V4.0

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